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Controlled Teleportation of an Arbitrary Two-Particle Pure or Mixed State by Using a Five-Qubit Cluster State

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Abstract A new scheme for controlled teleportation of an arbitrary two-particle pure or mixed state with the help of a five-qubit cluster state is proposed in detail. In this scheme, a five-particle cluster state is shared by a sender, a controller and a receiver. At first, the sender performs a four-qubit von-Neumann measurement on the qubits at hand, and the controller performs a Hadamard measurement on his qubit. Then the receiver can reconstruct the arbitrary two-particle pure or mixed state by performing some appropriate unitary transformations on his particles after he knows the measure results of the sender and the controller. This controlled teleportation scheme is deterministic.

Keywords Controlled teleportation · Cluster state · Arbitrary two-particle pure or mixed state

1 Introduction

Quantum teleportation is a technique for transfer of information between parties, using a distributed entangled state and a classical communication channel. Since Bennett et al. [1] presented the protocol of quantum teleportation in 1993, it has become one of the most important fields of quantum information. Several authors have devised protocols for the teleportation by using quantum entangled channels [2–8]. The first controlled teleportation protocol was proposed in 1998 by using a three-qubit Greenberger-Horne-Zeilinger (GHZ) state [9]. The basic idea of controlled teleportation is to transport an unknown quantum

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state with a controller, and it has been under the extensive investigations [10–19]. In these schemes the controlled teleportation of an unknown qubit state is through a three-qubit GHZ state or W state as the quantum channel along with classical communication. However, the entanglement in multi-qubit case is more complicated than in three-qubit case. Briegel and Raussendorf [20] showed that cluster states have some particular characters in the case of $N > 3$. For instance, the cluster state is maximally connected and its persistency is better than one of GHZ-class state. In other words, the cluster state has the properties of both the GHZ-class and the W-class entangled states, and is harder to destroy by local operations than GHZ-class states [21, 22]. In recent years, many teleportation protocols are proposed using the multi-particle cluster state as quantum channel. Nie et al. proposed a scheme of non-maximally entangled controlled teleportation using a four-particle cluster state [23]. Zhang and Liu proposed an economic and deterministic quantum teleportation of an arbitrary bipartite state using a four particles cluster state as quantum channel [24]. It has been shown that the four and five particles cluster states are important resources for teleportation of an arbitrary two particles state [25].

In this paper, we proposed a new scheme in detail for the controlled teleportation of an arbitrary two-particle pure or mixed state with the help of a five-qubit cluster state. In this scheme, a five-particle cluster state is prepared and shared by a sender, a controller and a receiver. The sender first performs a four-qubit von-Neumann measurement on her qubits under a complete basis of four-particle entangled states. Then the controller performs a Hadamard measurement on his qubit. Finally the receiver performs some appropriate unitary transformations on his qubits according to the measure results of the sender and the controller. Thus the task of controlled teleportation of an arbitrary bipartite state is completed. In Sect. 2, we investigate the controlled teleportation of an arbitrary bipartite pure state, and in Sect. 3, furthermore, we discuss the controlled teleportation of an arbitrary bipartite mixed state.

2 Controlled Teleportation of an Arbitrary Bipartite Pure State

We first prepare a cluster state with five particles 1, 2, 3, 4 and 5 [20]

$$|\Psi\rangle_{12345} = \frac{1}{2}(|00000\rangle + |00111\rangle + |11101\rangle + |11010\rangle)_{12345}, \tag{1}$$

where the particles 1 and 5 are kept by the sender Alice, the particle 3 by the controller Charlie, and the particles 2 and 4 by the receiver Bob. Now the sender Alice wants to teleport an arbitrary bipartite pure state to Bob. This arbitrary bipartite state can be usually written as

$$|\Phi\rangle_{xy} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \varepsilon|11\rangle, \tag{2}$$

where $\alpha^2 + \beta^2 + \gamma^2 + \varepsilon^2 = 1$. The state of the composite quantum system is

$$\begin{aligned}
|\Psi\rangle_{xy12345} &= |\Phi\rangle_{xy} \otimes |\Psi\rangle_{12345} \\
&= \frac{1}{2}(\alpha|0000000\rangle + \beta|0100000\rangle + \gamma|1000000\rangle + \varepsilon|1100000\rangle \\
&\quad + \alpha|0000111\rangle + \beta|0100111\rangle + \gamma|1000111\rangle + \varepsilon|1100111\rangle \\
&\quad + \alpha|0011101\rangle + \beta|0111101\rangle + \gamma|1011101\rangle + \varepsilon|1111101\rangle \\
&\quad + \alpha|0011010\rangle + \beta|0111010\rangle + \gamma|1011010\rangle + \varepsilon|1111010\rangle)_{xy12345}.
\end{aligned} \tag{3}$$

First, Alice performs a four-qubit von-Neumann measurement on her qubits x, y, 1 and 5 under a complete basis of four-particle entangled states. This complete basis is given by

$$|\Psi^1\rangle = \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle),$$

$$|\Psi^2\rangle = \frac{1}{2}(|0000\rangle + |0101\rangle - |1010\rangle - |1111\rangle),$$

$$|\Psi^3\rangle = \frac{1}{2}(|0000\rangle - |0101\rangle - |1010\rangle + |1111\rangle),$$

$$|\Psi^4\rangle = \frac{1}{2}(|0000\rangle - |0101\rangle + |1010\rangle - |1111\rangle),$$

$$|\Psi^5\rangle = \frac{1}{2}(|0010\rangle + |0111\rangle + |1000\rangle + |1101\rangle),$$

$$|\Psi^6\rangle = \frac{1}{2}(|0010\rangle + |0111\rangle - |1000\rangle - |1101\rangle),$$

$$|\Psi^7\rangle = \frac{1}{2}(|0010\rangle - |0111\rangle - |1000\rangle + |1101\rangle),$$

$$|\Psi^8\rangle = \frac{1}{2}(|0010\rangle - |0111\rangle + |1000\rangle - |1101\rangle),$$

$$|\Psi^9\rangle = \frac{1}{2}(|0001\rangle + |0100\rangle + |1011\rangle + |1110\rangle),$$

$$|\Psi^{10}\rangle = \frac{1}{2}(|0001\rangle + |0100\rangle - |1011\rangle - |1110\rangle),$$

$$|\Psi^{11}\rangle = \frac{1}{2}(|0001\rangle - |0100\rangle - |1011\rangle + |1110\rangle),$$

$$|\Psi^{12}\rangle = \frac{1}{2}(|0001\rangle - |0100\rangle + |1011\rangle - |1110\rangle),$$

$$|\Psi^{13}\rangle = \frac{1}{2}(|0011\rangle + |0110\rangle + |1001\rangle + |1100\rangle),$$

$$|\Psi^{14}\rangle = \frac{1}{2}(|0011\rangle + |0110\rangle - |1001\rangle - |1100\rangle),$$

$$|\Psi^{15}\rangle = \frac{1}{2}(|0011\rangle - |0110\rangle - |1001\rangle + |1100\rangle),$$

$$|\Psi^{16}\rangle = \frac{1}{2}(|0011\rangle - |0110\rangle + |1001\rangle - |1100\rangle),$$

which have 16 kinds of possible measure results $|\Psi^i\rangle$ $(i = 1, 2, \ldots, 16)$ by Alice with equal probability 1/16. There are also 16 kinds of corresponding collapse states $|\Phi^i\rangle_{234}$ $(i = 1, 2, \ldots, 16)$ of qubits 2, 3 and 4 after the measurement by Alice. The possible results of the measurement performed by Alice and the corresponding states obtained by Charlie and Bob are shown in Table 1.

Then Alice tells the result of her measurement to Bob and Charlie via a classical channel. If the controller Charlie allows Bob to get the initial state that Alice wants to send to Bob, then Charlie performs a measurement on his qubit 3 under the basis $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$,

Table 1 Alice's possibly measure results on qubits x, y, 1, 5 and the corresponding states obtained by Charlie and Bob, where the normalization factors have been omitted for convenience

Alice's measure result $\|\Psi^i\rangle$	$\|\Phi^i\rangle_{234}$ The corresponding states obtained by Charlie and Bob
$\|\Psi^1\rangle$	$\|\Phi^1\rangle_{234} = \alpha\|000\rangle + \beta\|011\rangle + \gamma\|101\rangle + \varepsilon\|110\rangle$
$\|\Psi^2\rangle$	$\|\Phi^2\rangle_{234} = \alpha\|000\rangle + \beta\|011\rangle - \gamma\|101\rangle - \varepsilon\|110\rangle$
$\|\Psi^3\rangle$	$\|\Phi^3\rangle_{234} = \alpha\|000\rangle - \beta\|011\rangle - \gamma\|101\rangle + \varepsilon\|110\rangle$
$\|\Psi^4\rangle$	$\|\Phi^4\rangle_{234} = \alpha\|000\rangle - \beta\|011\rangle + \gamma\|101\rangle - \varepsilon\|110\rangle$
$\|\Psi^5\rangle$	$\|\Phi^5\rangle_{234} = \alpha\|101\rangle + \beta\|110\rangle + \gamma\|000\rangle + \varepsilon\|011\rangle$
$\|\Psi^6\rangle$	$\|\Phi^6\rangle_{234} = \alpha\|101\rangle + \beta\|110\rangle - \gamma\|000\rangle - \varepsilon\|011\rangle$
$\|\Psi^7\rangle$	$\|\Phi^7\rangle_{234} = \alpha\|101\rangle - \beta\|110\rangle - \gamma\|000\rangle + \varepsilon\|011\rangle$
$\|\Psi^8\rangle$	$\|\Phi^8\rangle_{234} = \alpha\|101\rangle - \beta\|110\rangle + \gamma\|000\rangle - \varepsilon\|011\rangle$
$\|\Psi^9\rangle$	$\|\Phi^9\rangle_{234} = \alpha\|011\rangle + \beta\|000\rangle + \gamma\|110\rangle + \varepsilon\|101\rangle$
$\|\Psi^{10}\rangle$	$\|\Phi^{10}\rangle_{234} = \alpha\|011\rangle + \beta\|000\rangle - \gamma\|110\rangle - \varepsilon\|101\rangle$
$\|\Psi^{11}\rangle$	$\|\Phi^{11}\rangle_{234} = \alpha\|011\rangle - \beta\|000\rangle - \gamma\|110\rangle + \varepsilon\|101\rangle$
$\|\Psi^{12}\rangle$	$\|\Phi^{12}\rangle_{234} = \alpha\|011\rangle - \beta\|000\rangle + \gamma\|110\rangle - \varepsilon\|101\rangle$
$\|\Psi^{13}\rangle$	$\|\Phi^{13}\rangle_{234} = \alpha\|110\rangle + \beta\|101\rangle + \gamma\|011\rangle + \varepsilon\|000\rangle$
$\|\Psi^{14}\rangle$	$\|\Phi^{14}\rangle_{234} = \alpha\|110\rangle + \beta\|101\rangle - \gamma\|011\rangle - \varepsilon\|000\rangle$
$\|\Psi^{15}\rangle$	$\|\Phi^{15}\rangle_{234} = \alpha\|110\rangle - \beta\|101\rangle - \gamma\|011\rangle + \varepsilon\|000\rangle$
$\|\Psi^{16}\rangle$	$\|\Phi^{16}\rangle_{234} = \alpha\|110\rangle - \beta\|101\rangle + \gamma\|011\rangle - \varepsilon\|000\rangle$

and tells Bob about his result via a classical channel. The possible result of the measurement performed by Charlie and the corresponding states obtained by Bob are shown in Table 2.

Finally, Bob needs to perform an appropriate unitary transformation on his qubits 2 and 4 according to the results of Alice and Charlie so as to obtain the initial state that Alice wants to send to him. These unitary transformations are shown in Table 3.

3 Controlled Teleportation of an Arbitrary Bipartite Mixed State

With the same procedures as discussed in Sect. 2, we can realize the controlled teleportation of an arbitrary bipartite mixed state. The arbitrary bipartite mixed state ρ of system xy can be decomposed as [24, 26, 27]

$$\rho_{xy} = \sum_{j=1}^{4} p_j |\Phi_j\rangle_{xyxy}\langle\Phi_j|, \tag{4}$$

where $0 \le p_j \le 1$, $\sum_{j=1}^{4} p_j = 1$, $|\Phi_j\rangle_{xy} = \alpha_j|00\rangle + \beta_j|01\rangle + \gamma_j|10\rangle + \varepsilon_j|11\rangle$ are bipartite pure states with the same entanglement of formation as ρ and $\alpha_j^2 + \beta_j^2 + \gamma_j^2 + \varepsilon_j^2 = 1$ ($j = 1, 2, 3, 4$).

Firstly, Alice performs a four-qubit von-Neumann measurement on her qubits x, y, 1 and 5 under a complete basis of four-particle entangled states. The result of her measurement is one of $|\Psi^i\rangle$ ($i = 1, 2, \dots, 16$) with equal probability $1/16$. While the corresponding collapse mixed state obtained by Charlie and Bob is one of the ρ_{234}^i ($i = 1, 2, \dots, 16$)

$$\rho_{234}^i = \sum_{j=1}^{4} p_j |\Phi_j^i\rangle_{234234}\langle\Phi_j^i| \quad (i = 1, 2, \dots, 16), \tag{5}$$

Table 2 Charlie's possibly measure results on qubit 3 and the corresponding states obtained by Bob, where the normalization factors have been omitted for convenience

Measure result of Charlie	$\lvert\Psi^i\rangle_{24}^-$ the states obtained by Bob	Measure result of Charlie	$\lvert\Psi^i\rangle_{24}^+$ the states obtained by Bob
$\lvert-\rangle_3^1$	$\lvert\Psi^1\rangle_{24}^- = \alpha\lvert 00\rangle - \beta\lvert 01\rangle + \gamma\lvert 11\rangle - \varepsilon\lvert 10\rangle$	$\lvert+\rangle_3^1$	$\lvert\Psi^1\rangle_{24}^+ = \alpha\lvert 00\rangle + \beta\lvert 01\rangle + \gamma\lvert 11\rangle + \varepsilon\lvert 10\rangle$
$\lvert-\rangle_3^2$	$\lvert\Psi^2\rangle_{24}^- = \alpha\lvert 00\rangle - \beta\lvert 01\rangle - \gamma\lvert 11\rangle + \varepsilon\lvert 10\rangle$	$\lvert+\rangle_3^2$	$\lvert\Psi^2\rangle_{24}^+ = \alpha\lvert 00\rangle + \beta\lvert 01\rangle - \gamma\lvert 11\rangle - \varepsilon\lvert 10\rangle$
$\lvert-\rangle_3^3$	$\lvert\Psi^3\rangle_{24}^- = \alpha\lvert 00\rangle + \beta\lvert 01\rangle - \gamma\lvert 11\rangle - \varepsilon\lvert 10\rangle$	$\lvert+\rangle_3^3$	$\lvert\Psi^3\rangle_{24}^+ = \alpha\lvert 00\rangle - \beta\lvert 01\rangle - \gamma\lvert 11\rangle + \varepsilon\lvert 10\rangle$
$\lvert-\rangle_3^4$	$\lvert\Psi^4\rangle_{24}^- = \alpha\lvert 00\rangle + \beta\lvert 01\rangle + \gamma\lvert 11\rangle + \varepsilon\lvert 10\rangle$	$\lvert+\rangle_3^4$	$\lvert\Psi^4\rangle_{24}^+ = \alpha\lvert 00\rangle - \beta\lvert 01\rangle + \gamma\lvert 11\rangle - \varepsilon\lvert 10\rangle$
$\lvert-\rangle_3^5$	$\lvert\Psi^5\rangle_{24}^- = \alpha\lvert 11\rangle - \beta\lvert 10\rangle + \gamma\lvert 00\rangle - \varepsilon\lvert 01\rangle$	$\lvert+\rangle_3^5$	$\lvert\Psi^5\rangle_{24}^+ = \alpha\lvert 11\rangle + \beta\lvert 10\rangle + \gamma\lvert 00\rangle + \varepsilon\lvert 01\rangle$
$\lvert-\rangle_3^6$	$\lvert\Psi^6\rangle_{24}^- = \alpha\lvert 11\rangle - \beta\lvert 10\rangle - \gamma\lvert 00\rangle + \varepsilon\lvert 01\rangle$	$\lvert+\rangle_3^6$	$\lvert\Psi^6\rangle_{24}^+ = \alpha\lvert 11\rangle + \beta\lvert 10\rangle - \gamma\lvert 00\rangle - \varepsilon\lvert 01\rangle$
$\lvert-\rangle_3^7$	$\lvert\Psi^7\rangle_{24}^- = \alpha\lvert 11\rangle + \beta\lvert 10\rangle - \gamma\lvert 00\rangle - \varepsilon\lvert 01\rangle$	$\lvert+\rangle_3^7$	$\lvert\Psi^7\rangle_{24}^+ = \alpha\lvert 11\rangle - \beta\lvert 10\rangle - \gamma\lvert 00\rangle + \varepsilon\lvert 01\rangle$
$\lvert-\rangle_3^8$	$\lvert\Psi^8\rangle_{24}^- = \alpha\lvert 11\rangle + \beta\lvert 10\rangle + \gamma\lvert 00\rangle + \varepsilon\lvert 01\rangle$	$\lvert+\rangle_3^8$	$\lvert\Psi^8\rangle_{24}^+ = \alpha\lvert 11\rangle - \beta\lvert 10\rangle + \gamma\lvert 00\rangle - \varepsilon\lvert 01\rangle$
$\lvert-\rangle_3^9$	$\lvert\Psi^9\rangle_{24}^- = -\alpha\lvert 01\rangle + \beta\lvert 00\rangle - \gamma\lvert 10\rangle + \varepsilon\lvert 11\rangle$	$\lvert+\rangle_3^9$	$\lvert\Psi^9\rangle_{24}^+ = \alpha\lvert 01\rangle + \beta\lvert 00\rangle + \gamma\lvert 10\rangle + \varepsilon\lvert 11\rangle$
$\lvert-\rangle_3^{10}$	$\lvert\Psi^{10}\rangle_{24}^- = -\alpha\lvert 01\rangle + \beta\lvert 00\rangle + \gamma\lvert 10\rangle - \varepsilon\lvert 11\rangle$	$\lvert+\rangle_3^{10}$	$\lvert\Psi^{10}\rangle_{24}^+ = \alpha\lvert 01\rangle + \beta\lvert 00\rangle - \gamma\lvert 10\rangle - \varepsilon\lvert 11\rangle$
$\lvert-\rangle_3^{11}$	$\lvert\Psi^{11}\rangle_{24}^- = -\alpha\lvert 01\rangle - \beta\lvert 00\rangle + \gamma\lvert 10\rangle + \varepsilon\lvert 11\rangle$	$\lvert+\rangle_3^{11}$	$\lvert\Psi^{11}\rangle_{24}^+ = \alpha\lvert 01\rangle - \beta\lvert 00\rangle - \gamma\lvert 10\rangle + \varepsilon\lvert 11\rangle$
$\lvert-\rangle_3^{12}$	$\lvert\Psi^{12}\rangle_{24}^- = -\alpha\lvert 01\rangle - \beta\lvert 00\rangle - \gamma\lvert 10\rangle - \varepsilon\lvert 11\rangle$	$\lvert+\rangle_3^{12}$	$\lvert\Psi^{12}\rangle_{24}^+ = \alpha\lvert 01\rangle - \beta\lvert 00\rangle + \gamma\lvert 10\rangle - \varepsilon\lvert 11\rangle$
$\lvert-\rangle_3^{13}$	$\lvert\Psi^{13}\rangle_{24}^- = -\alpha\lvert 10\rangle + \beta\lvert 11\rangle - \gamma\lvert 01\rangle + \varepsilon\lvert 00\rangle$	$\lvert+\rangle_3^{13}$	$\lvert\Psi^{13}\rangle_{24}^+ = \alpha\lvert 10\rangle + \beta\lvert 11\rangle + \gamma\lvert 01\rangle + \varepsilon\lvert 00\rangle$
$\lvert-\rangle_3^{14}$	$\lvert\Psi^{14}\rangle_{24}^- = -\alpha\lvert 10\rangle + \beta\lvert 11\rangle + \gamma\lvert 01\rangle - \varepsilon\lvert 00\rangle$	$\lvert+\rangle_3^{14}$	$\lvert\Psi^{14}\rangle_{24}^+ = \alpha\lvert 10\rangle + \beta\lvert 11\rangle - \gamma\lvert 01\rangle - \varepsilon\lvert 00\rangle$
$\lvert-\rangle_3^{15}$	$\lvert\Psi^{15}\rangle_{24}^- = -\alpha\lvert 10\rangle - \beta\lvert 11\rangle + \gamma\lvert 01\rangle + \varepsilon\lvert 00\rangle$	$\lvert+\rangle_3^{15}$	$\lvert\Psi^{15}\rangle_{24}^+ = \alpha\lvert 10\rangle - \beta\lvert 11\rangle - \gamma\lvert 01\rangle + \varepsilon\lvert 00\rangle$
$\lvert-\rangle_3^{16}$	$\lvert\Psi^{16}\rangle_{24}^- = -\alpha\lvert 10\rangle - \beta\lvert 11\rangle - \gamma\lvert 01\rangle - \varepsilon\lvert 00\rangle$	$\lvert+\rangle_3^{16}$	$\lvert\Psi^{16}\rangle_{24}^+ = \alpha\lvert 10\rangle - \beta\lvert 11\rangle + \gamma\lvert 01\rangle - \varepsilon\lvert 00\rangle$

Table 3 The correspondingly unitary transformation performed by Bob according to the state obtained by Bob

$\lvert\Psi^i\rangle_{24}^-$	U_{24}^{i-}	$\lvert\Psi^i\rangle_{24}^-$	U_{24}^{i-}	$\lvert\Psi^i\rangle_{24}^+$	U_{24}^{i+}	$\lvert\Psi^i\rangle_{24}^+$	U_{24}^{i+}
$\lvert\Psi^1\rangle_{24}^-$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\lvert\Psi^9\rangle_{24}^-$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\lvert\Psi^1\rangle_{24}^+$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\lvert\Psi^9\rangle_{24}^+$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
$\lvert\Psi^2\rangle_{24}^-$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\lvert\Psi^{10}\rangle_{24}^-$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\lvert\Psi^2\rangle_{24}^+$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\lvert\Psi^{10}\rangle_{24}^+$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$
$\lvert\Psi^3\rangle_{24}^-$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\lvert\Psi^{11}\rangle_{24}^-$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\lvert\Psi^3\rangle_{24}^+$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\lvert\Psi^{11}\rangle_{24}^+$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
$\lvert\Psi^4\rangle_{24}^-$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\lvert\Psi^{12}\rangle_{24}^-$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\lvert\Psi^4\rangle_{24}^+$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\lvert\Psi^{12}\rangle_{24}^+$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$
$\lvert\Psi^5\rangle_{24}^-$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^{13}\rangle_{24}^-$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^5\rangle_{24}^+$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^{13}\rangle_{24}^+$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

Table 3 (*Continued*)

$\lvert\Psi^i\rangle_{24}^-$	U_{24}^{i-}	$\lvert\Psi^i\rangle_{24}^-$	U_{24}^{i-}	$\lvert\Psi^i\rangle_{24}^+$	U_{24}^{i+}	$\lvert\Psi^i\rangle_{24}^+$	U_{24}^{i+}
$\lvert\Psi^6\rangle_{24}^-$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^{14}\rangle_{24}^-$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^6\rangle_{24}^+$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^{14}\rangle_{24}^+$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$
$\lvert\Psi^7\rangle_{24}^-$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^{15}\rangle_{24}^-$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^7\rangle_{24}^+$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^{15}\rangle_{24}^+$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$
$\lvert\Psi^8\rangle_{24}^-$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^{16}\rangle_{24}^-$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^8\rangle_{24}^+$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\lvert\Psi^{16}\rangle_{24}^+$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$

where $|\Phi_j^i\rangle_{234}$ has the same form as $|\Phi^i\rangle_{234}$ in Table 1, and the only difference is to replace the coefficients α, β, γ and ε with $\alpha_j, \beta_j, \gamma_j$ and ε_j. Then Alice tells the result of her measurement to Bob and Charlie via a classical channel. If the controller Charlie allows Bob to get the initial state that Alice wants to send to Bob, then Charlie performs a measurement on his qubit 3 under the basis $\{|+\rangle, |-\rangle\}$, and tells the result of his measurement to Bob via a classical channel. The state with the particles 2 and 4 which are kept in Bob's hand would be collapsed onto one of the following states

$$\rho_{24}^{i-} = \sum_{j=1}^{4} p_j |\Psi_j^i\rangle_{24\,24}^{-\,-}\langle\Psi_j^i| \quad (i = 1, 2, \ldots, 16), \tag{6}$$

or

$$\rho_{24}^{i+} = \sum_{j=1}^{4} p_j |\Psi_j^i\rangle_{24\,24}^{+\,+}\langle\Psi_j^i| \quad (i = 1, 2, \ldots, 16). \tag{7}$$

where $|\Psi_j^i\rangle_{24}^-$ or $|\Psi_j^i\rangle_{24}^+$ has the same form as $|\Psi^i\rangle_{24}^-$ or $|\Psi^i\rangle_{24}^+$ in Table 2, and the only difference is to replace the coefficients α, β, γ and ε with $\alpha_j, \beta_j, \gamma_j$ and ε_j. In order to reconstruct the initial bipartite mixed state, Consequently, Bob needs to perform an appropriate unitary transformation (U_{24}^{i-} or U_{24}^{i+}), $i = 1, 2, \ldots, 16$, which are shown in Table 3) on his qubits 2 and 4 (ρ_{24}^{i-} or ρ_{24}^{i+}) according to the results of Alice and Charlie so that he obtained the initial mixed state that Alice wants to send to him. Thus the task of controlled teleportation of an arbitrary bipartite mixed state is completed.

In fact, if we regard the controlled teleportation of a mixed state as the "compressed density matrix" [28], then the controlled teleportation of the mixed state would reduce to the controlled teleportation of an ensemble of pure states, which could also be equal to the sum of individual controlled teleportations of pure states using the above method [29].

4 Summary

In this paper, we proposed the new scheme in detail for controlled teleportation of an arbitrary two-particle pure and mixed state by using the properties of five-qubit cluster state. In this scheme, a five-particle cluster state is prepared and shared by a sender, a controller and a receiver. The sender first performs a four-qubit von-Neumann measurement on her qubits under a complete basis of four-particle entangled states, then the controller performs a measurement on his qubit under the basis $\{|+\rangle, |-\rangle\}$. Finally the receiver performs some appropriate unitary transformation on his qubits according to the measure results from the sender and the controller. Thus the task of controlled teleportation of an arbitrary bipartite state is completed. And the probability of successful controlled teleportation is 100%. Without the cooperation of controller, the receiver cannot reconstruct the initial arbitrary two-particle state by himself. This property of the scheme can be utilized to construct a controlled quantum channel, which may be useful in the future quantum computation.

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